

UNCLASSIFIED

AD NUMBER
AD343598
CLASSIFICATION CHANGES
TO: unclassified
FROM: restricted
LIMITATION CHANGES
TO: Approved for public release, distribution unlimited
FROM: Controlling Organization: British Embassy, 3100 Massachusetts Avenue, NW, Washington, DC 20008.
AUTHORITY
DSTL, ADM 204/2431, 19 May 2009; DSTL, ADM 204/2431, 19 May 2009

THIS PAGE IS UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not .. be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

NOTICE:

THIS DOCUMENT CONTAINS INFORMATION
AFFECTING THE NATIONAL DEFENSE OF
THE UNITED STATES WITHIN THE MEAN-
ING OF THE ESPIONAGE LAWS, TITLE 18,
U.S.C., SECTIONS 793 and 794. THE
TRANSMISSION OR THE REVELATION OF
ITS CONTENTS IN ANY MANNER TO AN
UNAUTHORIZED PERSON IS PROHIBITED
BY LAW.

TIAL
CRIZED

CATALOGED BY DDC
AD No. 343598

RESTRICTED

ACSIL/ADM/53/251

CONFIDENTIAL - Modified Handling Authorized Copy No. 13
U.K. RESTRICTED



A.R.L/M/N2

THE DOPPLER-TOLERANCE OF A LINEARLY
FREQUENCY-MODULATED PULSE [U]

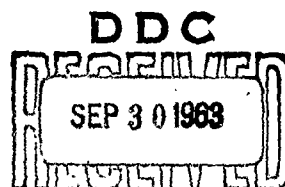
BY

E. M. WILSON

MAY 1963

IMPORTANT

Attention is drawn to
the provisions of the
Official Secrets Acts



ADMIRALTY RESEARCH LABORATORY

TEDDINGTON MIDDLESEX

U.S. CONFIDENTIAL - Modified Handling Authorized
U.K. RESTRICTED

CONDITIONS OF RELEASE

1. This information is disclosed only for official use by the recipient Government and such of its contractors, under seal of secrecy, as may be engaged on a defence project. Disclosure to any other Government or release to the Press or in any other way would be a breach of these conditions.
 2. The information should be safeguarded under rules designed to give the same standard of security as that maintained by Her Majesty's Government in the United Kingdom.
- The recipient is warned that information contained in this document may be subject to privately-owned rights.

EXCLUDED FROM AUTOMATIC
REGRADING, Doc Dir 5200.10
DOES NOT APPLY

TIAL
CRIZED

THE DOPPLER-TOLERANCE OF A LINEARLY
FREQUENCY-MODULATED PULSE

by

E. M. Wilson

ABSTRACT

A theoretical analysis is made of the reduction due to the Doppler effect in the correlation between a linearly frequency-modulated pulse and its echo from a moving target. The results agree very closely with numerical calculations made on a different basis, and show that such a pulse can be highly doppler-tolerant.

"This document contains information affecting the National Defense of the United States within the meaning of the Espionage Laws, Title 18, U. S. C., Section 793 and 794. Its transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law."

U. S. CONFIDENTIAL
MODIFIED HANDLING AUTHORIZED

A linearly frequency-modulated (or L.F.M.) pulse of duration T and bandwidth B can be described by the equation

$$y_1(t) = \sin \left\{ 2\pi \left(f_1 + \frac{B}{2T} t \right) t + \phi_1 \right\}, \quad 0 \leq t \leq T \quad (1)$$

in which ϕ_1 is merely a phase constant. At time t the instantaneous frequency is equal to

$$\frac{d}{dt} \left[f_1 t + \frac{B}{2T} t^2 \right], \quad \text{i.e.} \quad f_1 + \frac{B}{T} t \quad (2)$$

We imagine such a pulse to be reflected by a target and the echo, on arrival at the point of transmission, to be correlated with the original pulse, appropriately delayed. The delay which produces maximum correlation serves to estimate the range at which reflection occurred.

If the target is moving, the pulse suffers a frequency shift (the Doppler effect) which, since the pulse is linearly frequency-modulated, has almost the same appearance as an alteration of range, in that the delay producing maximum correlation between transmitted and received pulses is changed. In addition, however, there is a reduction in the size of the correlation peak. The present paper evaluates this reduction. For the sake of greater generality we suppose the pulse to be heterodyned up by a frequency F before transmission, its equation becoming

$$y_2(t) = \sin \left\{ 2\pi \left(f_1 + F + \frac{B}{2T} t \right) t + \phi_2 \right\}, \quad (3)$$

and the echo to be heterodyned down by the same frequency before correlation. Thus reflection, with the attendant Doppler effect, and correlation take place over different frequency bands.

The equation of the pulse just after reflection may be written

$$\gamma_2(t) = \sin \left\{ 2\pi \left(f_1 + F + \frac{B}{2T} \frac{c+v}{c-v} \cdot t \right) \frac{c+v}{c-v} t + \alpha_2 \right\}, \quad (4)$$

$$0 \leq t \leq \frac{c-v}{c+v} T$$

$$= \sin \left\{ 2\pi \left[f_1 + F + \frac{B}{2T} (1+A) t \right] (1+A) t + \alpha_2 \right\}, \quad (5)$$

$$0 \leq t \leq T/(1+A)$$

where c is the velocity of the pulse, v is the component of velocity of the target towards the transmitter, and A denotes $2v/(c-v)$. In each of the equations above, the time origin is taken at the beginning of the pulse. On heterodyning down we have, for comparison with the original pulse,

$$\gamma_4(t) = \sin \left\{ 2\pi \left[f_1 + F + \frac{B}{2T} (1+A) t \right] (1+A) t - 2\pi F t + \alpha_3 \right\} \quad (6)$$

After elimination of the time difference due to travel of the pulse, the correlation of (6), delayed relatively by a time $\tau > 0$, with (1) is either

$$\frac{2}{T-\tau} \int_{\tau}^T \gamma_1(t) \gamma_4(t-\tau) dt \quad (7)$$

or

$$\frac{2(c+v)}{(c-v)T} \int_{\tau}^{\tau + \frac{c-v}{c+v} T} \gamma_1(t) \gamma_4(t-\tau) dt \quad (7a)$$

according to which upper limit is the lesser. At this point we shall take (7) to be the appropriate form, which is equivalent to restricting our view to values of τ greater than $AT/(1+A)$. It will appear later (see equation (24)) that when the correlation function is near its maximum τ will in fact satisfy this restriction.

U. S. CONFIDENTIAL
MODIFIED HATHORN AUTHORIZED

The product of two sines in (7) can be expressed as the difference of two cosines, the integral of one of which, representing the sum frequency, can be neglected in comparison with the integral of the other, for any likely practical values of the parameters. We are then left with

$$\frac{1}{T-\tau} \int_{\tau}^T \cos q(t) dt \quad (8)$$

where

$$q(t) = 2\pi \left[f_1 + F + \frac{B}{2T}(1+k)(t-\tau) \right] (1+k)(t-\tau) - 2\pi F(t-\tau) - 2\pi \left(f_1 + \frac{B}{2T}t \right) t + \phi_2 - \phi_1 \quad (9)$$

$$= 2\pi \left[A_0 + A_1 t + \frac{KB}{T} t^2 \right] \quad (10)$$

Because of the phase changes introduced by heterodyning, A_0 must be regarded as an arbitrary unknown. The other coefficients in (10) are given by

$$A_1 = k(f_1 + F) - \frac{B}{T}(1+k)^2 \tau \quad (11)$$

and

$$K = k + \frac{1}{2}k^2 = \frac{2vc}{(c-v)^2} \quad (12)$$

Both k and $\frac{B}{T}$ will in practice be small, and the right hand sides of (11) and (12) will be approximately

$$k(f_1 + F) - \frac{B}{T} \tau \quad (13)$$

and

$$k \left(\approx \frac{2v}{c} \right) \quad (14)$$

respectively.

U. S. CONFIDENTIAL
MODIFIED HATHORN AUTHORIZED

Now

$$\begin{aligned} & \int_{\tau}^T \cos 2\pi \left(A_0 + A_1 t + \frac{KB}{T} t^2 \right) dt \\ &= \int_{\tau}^T \cos 2\pi \left(\frac{KB}{T} \left[t + \frac{A_1 T}{2KB} \right]^2 + A_0 - \frac{A_1^2 T}{4KB} \right) dt \end{aligned} \quad (15)$$

$$\begin{aligned} &= \cos 2\pi \left(A_0 - \frac{A_1^2 T}{4KB} \right) \int_{u_1}^{u_2} \cos 2\pi \frac{KB}{T} u^2 du \\ &\quad - \sin 2\pi \left(A_0 - \frac{A_1^2 T}{4KB} \right) \int_{u_1}^{u_2} \sin 2\pi \frac{KB}{T} u^2 du \end{aligned} \quad (16)$$

where the limits of integration are linear functions of τ —

$$u_1 = \tau + \frac{A_1 T}{2KB} = \alpha_1 + \beta_1 \tau, \quad \text{say,} \quad (17)$$

$$\text{and} \quad u_2 = T + \frac{A_1 T}{2KB} = \alpha_2 + \beta_2 \tau \quad (18)$$

For fixed τ and variation of the unknown phase represented by A_0 ,
(16) has a maximum value of

$$M(\tau) = \sqrt{\left\{ \left[\int_{u_1}^{u_2} \cos 2\pi \frac{KB}{T} u^2 du \right]^2 + \left[\int_{u_1}^{u_2} \sin 2\pi \frac{KB}{T} u^2 du \right]^2 \right\}} \quad (19)$$

(because $a \cos \theta + b \sin \theta = \sqrt{a^2 + b^2} \cos \left(\theta - \tan^{-1} \frac{b}{a} \right)$).

It will suffice to find the maximum of M as τ varies, since the factor $\frac{1}{T-\tau}$ varies too slowly, in comparison with M , to have much

effect on the value of τ associated with maximum correlation.

Differentiating (19) gives

$$\begin{aligned} \frac{dM}{d\tau} = \frac{1}{M} & \left\{ (A_2 \cos 2\pi \frac{KB}{T} u_2^2 - A_1 \cos 2\pi \frac{KB}{T} u_1^2) \int_{u_1}^{u_2} \cos 2\pi \frac{KB}{T} u^2 du \right. \\ & \left. + (A_2 \sin 2\pi \frac{KB}{T} u_2^2 - A_1 \sin 2\pi \frac{KB}{T} u_1^2) \int_{u_1}^{u_2} \sin 2\pi \frac{KB}{T} u^2 du \right\} \end{aligned} \quad (20)$$

Since, when k is small, β_1 and β_2 are large and nearly equal, being $-(2k+k^2)^{-1}$ and $-\{1+(2k+k^2)^{-1}\}$ respectively, (20) will vanish for a value of τ close to that, say τ_0 , which makes $u_1 = -u_2$. The maximum of M will therefore be close to

$$M(\tau_0) = 2 \left| \int_0^U e^{2\pi i \frac{KB}{T} u^2} du \right| \quad (21)$$

where
$$U = \frac{T - \tau_0}{2} \quad (22)$$

and
$$\tau_0 = \frac{T}{B} \left\{ \frac{(2k+k^2)B + 2k(f_1 + F)}{2 + 2k + k^2} \right\} \quad (23)$$

$$\approx \frac{T}{B} \left\{ B + f_1 + F \right\} \frac{k}{1+k} \quad (24)$$

The upper limit, U , of the integral in (21) is a multiple of T - say $U = \gamma T$ - and the integral can be written

$$\begin{aligned} & \int_0^{\gamma T} e^{2\pi i \frac{KB}{T} u^2} du \\ &= \sqrt{\frac{T}{4KB}} \int_0^{\gamma \sqrt{4KB}} e^{i \frac{\pi}{2} x^2} dx \end{aligned} \quad (25)$$

This is now in the standard form for Fresnel integrals.

The maximum possible value of the correlation (8) is thus approximately

$$\frac{1}{2\gamma T} \cdot 2 \sqrt{\frac{T}{4KB}} \left| \int_0^{\gamma \sqrt{4KB T}} e^{i \frac{1}{2} x^2} dx \right| \quad (26)$$

$$= \frac{1}{X} \left| C(X) + i S(X) \right| \quad (27)$$

where C and S are the usual Fresnel integrals, which can be obtained from tables (e.g. [1]),

$$X = 2\gamma \sqrt{KB T} \approx 2\gamma \sqrt{KB T} \quad (28)$$

and
$$2\gamma = 1 - \frac{B + S_1 + F}{B} \cdot \frac{A}{1+A} \quad (29)$$

Although γ will usually be fairly close to $\frac{1}{2}$ unless the bandwidth is small, it will not always be close enough for $\sqrt{KB T}$ to be an adequate approximation to X .

Figure 1 shows a graph of (27) against the velocity ratio $\frac{v}{c}$, for a case in which the ratio of maximum frequency to bandwidth (i.e. $\frac{B+S_1+F}{B}$) is 15, and the product BT is equal to 100. Also shown on the figure are three points obtained by a purely numerical process for correlating clipped functions, described in [2], the data being a set of about two hundred zero-crossings of a particular

L.F.M. pulse. These points lie, to graphical accuracy, on the theoretical curve just obtained (in fact, agreement is to about three decimal places). The vertical lines drawn downwards from the calculated points show the approximate extent of possible correlation loss in this particular case due to variation of the unknown phase change introduced on heterodyning.

Nothing whatever has so far been said about the effects of noise, although it is only when noise largely obscures the echo pulse that the correlation process is required. In order to detect any echo, correlation might be carried out continuously between a reference pulse and all possible received waveforms of the proper bandwidth and of the same duration as the reference. A form of correlation function closer to practice might then differ from (8) in having a factor $\frac{1}{T}$ outside the integral instead of $\frac{1}{T-\tau}$. We have already neglected variation of this factor in finding the position of maximum correlation and our results can, therefore, be transformed immediately to refer to the more practical situation simply by multiplying the maximum correlation by $\frac{T-\tau_a}{T}$. The broken curve in figure 1 shows the effect in that case. Although we have throughout supposed τ to be positive, that is, only approaching targets have been explicitly considered, we can expect that for small values of the velocity ratio $\frac{v}{c}$ our curves will apply to receding targets by reflection in the $v=0$ axis. This form of correlation, therefore, leads to an angular maximum in the doppler-tolerance curve.

CORRELATION WITH AN EXTENDED REFERENCE PULSE

It is quite possible to correlate the received echo not simply with the original pulse, but with an extended version of the original pulse, prolonged so that correlation is always effective for the full duration

of the echo, which is approximately T as long as the velocity ratio $\frac{v}{c}$ is small. The expression (7a), not (7), must then be taken as the correlation function for all values of v instead of only for those less than $kT/(1+k)$. Whether the factor outside the integral is taken as $\frac{2}{T}$ or as $\frac{c+v}{c-v} \cdot \frac{2}{T}$ is in this case not of much importance. Equation (21) remains relevant provided that the upper limit of the integral is replaced by

$$\frac{c-v}{c+v} \cdot \frac{T}{2} \quad \left(\approx \frac{T}{2} \right) \quad (30)$$

There is also a very small alteration in the delay producing maximum correlation, but the approximation (24) still holds. The principal result is that the maximum correlation will be given by

$$\frac{1}{Y} \left| C(Y) + i S(Y) \right| \quad (27a)$$

where $Y = \frac{1}{1+k} \sqrt{KBT} = \sqrt{K'BT}$ (31)

and $K' = \frac{k + \frac{1}{2}k^2}{1 + 2k + k^2} \approx k$ (32)

It is, therefore, an acceptable approximation to say that correlating with an extended reference pulse is equivalent to taking Y to be $\frac{1}{2}$. For use in this connection figure 2 shows

$$\frac{1}{\sqrt{K'BT}} \left| C(\sqrt{K'BT}) + i S(\sqrt{K'BT}) \right|$$

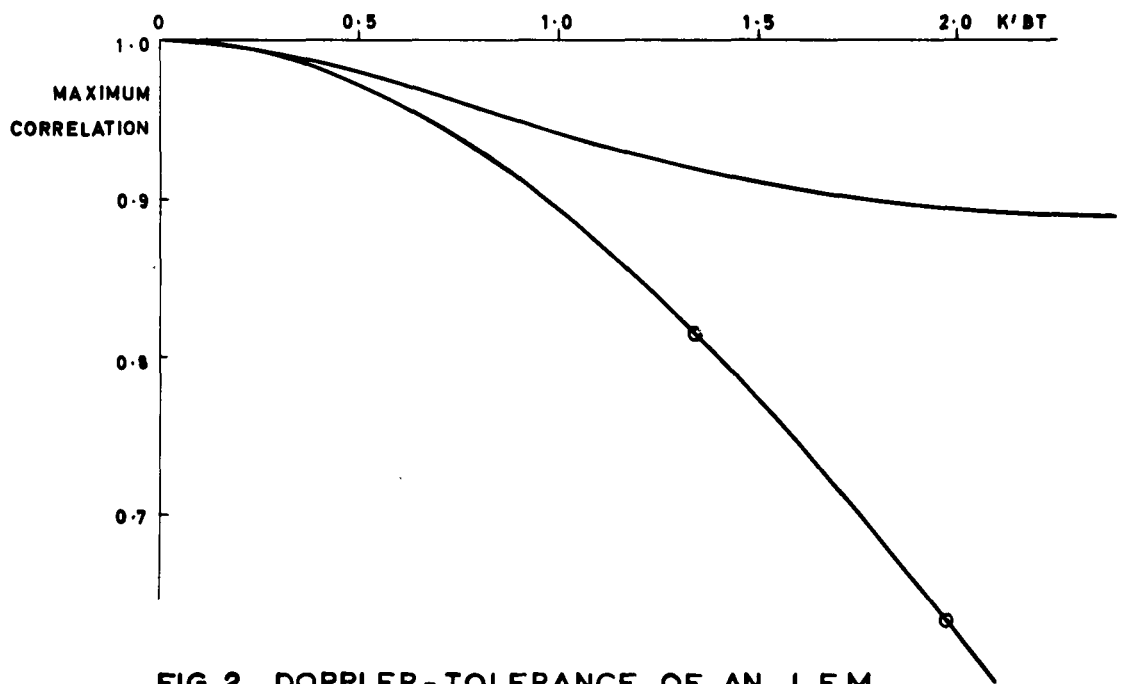
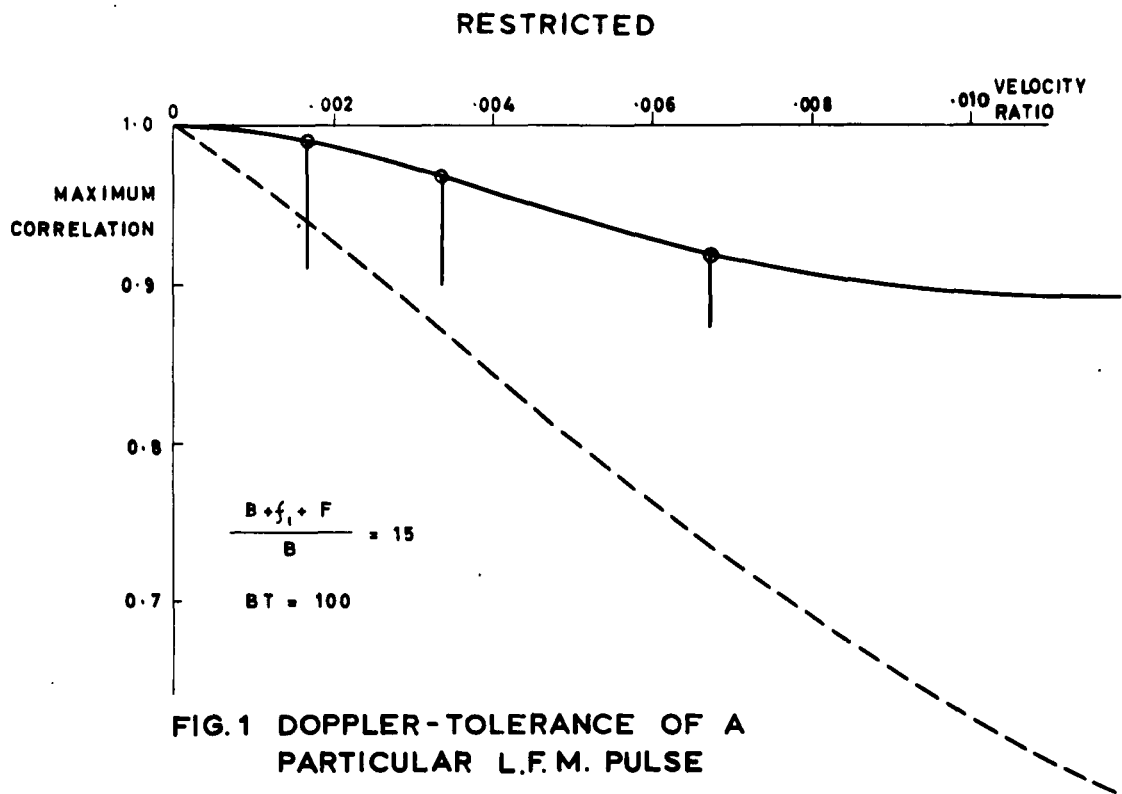
as a function of $K'\delta T$. The marked points on this graph represent check-values obtained by the process described in [2]. This curve may also be considered as a first approximation, independent of particular frequency, to the theoretical curves for the case of an unextended reference when the velocity ratio is very small. The curve shown in figure 1 is repeated on figure 2 for comparison.

E.M. Wilson (PSO)

EMW/BB

REFERENCES

1. A. Van WIJNGAARDEN and W. L. SCHEEN, Table of Fresnel Integrals,
report R49 of the Computation Department of
the Mathematical Centre at Amsterdam
2. E. M. WILSON ARL/M/R1, The Integration of Clipped Functions - I :
A Simple Correlation Process



RESTRICTED

DISTRIBUTION

	<u>Copy No.</u>
D.P.R.	1 - 2
D.R.D.S. (A.C.S.I.L. for distribution)	3 - 27
Scientific Adviser B.N.S. Washington.	28
D.O.R.	29
A.S.W.E.	30
A.E.W.	31
A.U.W.E.	32
S.E.R.L.	33
H.M.N.A.O.	34
Meteorological Office.	35
Dr. Hollingdale, R.A.E.	36
R.R.E.	37
A.W.R.E.	38
Dr. Howlett, A.E.R.E.	39
Dr. Goodwin, W.P.L.	40
A.R.L. File	41 - 50



*Information Centre
Knowledge Services*
[dstl] Porton Down,
Salisbury
Wiltshire
SP14 0JQ
22000-6218
Tel: 01980-613753
Fax: 01980-613976

Defense Technical Information Center (DTIC)
8725 John J. Kingman Road, Suit 0944
Fort Belvoir, VA 22060-6218
U.S.A.

AD#: AD0343598

Date of Search: 19 May 2009

Record Summary: ADM 204/2431

Title: Doppler tolerance of a linearly frequency modulated pulse
Availability Open Document, Open Description, Normal Closure before FOI Act: 30 years
Former reference (Department) M/N2
Held by The National Archives, Kew

This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (<http://www.nationalarchives.gov.uk>) and found the document is available and releasable to the public.

Access to UK public records is governed by statute, namely the Public Records Act, 1958, and the Public Records Act, 1967.
The document has been released under the 30 year rule.
(The vast majority of records selected for permanent preservation are made available to the public when they are 30 years old. This is commonly referred to as the 30 year rule and was established by the Public Records Act of 1967).

This document may be treated as UNLIMITED.